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Note on Land Use in a Long Narrow City*

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In their recent paper, Solow and Vickrey [2] show that there are two configurations of the long narrow city in an optimal allocation. One of these implies that the width of the road is constant. In elaborating the optimal configuration, however, Solow and Vickrey explicitly assume that the opportunity cost of land to the city as a whole is zero; they do not discuss the case when the latter is positive. The following comment shows that a constant width configuration is inefficient when the opportunity cost of land is positive.

Let the problem be defined as follows: Minimize

$$\int_0^L \left[v(x) f\left(\frac{v(x)}{w(x)}\right) + Rw(x)\right] dx,$$

subject to

$$Y(0) = 0,$$
$$Y(L) = A,$$

where

$$v(x) = 2gY(A - Y),$$
$$w(x) = W - Y',$$

$A, W, R =$ given parameters representing total business area, width of the city and opportunity costs of land, respectively.

Applying the Euler theorem RW is added to the lefthand side of Eq. (1) of Solow and Vickrey. Using the Caratheodory's "fundamental relations of the calculus of variation," it follows that the lefthand side of the modified equation must vanish, if the length of the city is optimal. Therefore, the original lefthand side of Eq. (1) must now be negative. This implies that the optimal road configuration must be the one discussed by

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Solow and Vickrey in their Section 3.b.(III); (it is reproduced in Fig. 1B). This characteristic of the land use is also implied by the model of Mills and De Ferranti [1] for a circular city when the opportunity cost of land is assigned the value of agricultural rent.

The economic interpretation is straightforward. For any given length of the city L, the value of C in Eq. (1) of Solow and Vickrey reflects the saving in the minimized aggregate transportation costs which can be realized per unit increment in the length of the city. Therefore, it reflects the marginal product of land in saving transportation costs, divided by the width of the city W. Hence, if C in Eq. (1) of Solow and Vickrey is zero, marginal product of land $(-C/W)$ is exceeded by its opportunity cost R. Resources can then be saved by reducing the length of the city until the marginal product of land $(-C/W)$ increases up to R. Thus, at the optimum, C of Eq. (1) of Solow and Vickrey becomes negative, which implies that the width of the road vanishes at the edges of the city.

If C is negative for a given L and its absolute value exceeds WR, the marginal product of land is higher than its opportunity cost and the city should be lengthened in order to be efficient. If C is smaller than WR, the length of the city should be reduced.

A constant width configuration is inefficient even when the length of the city is bounded from above (e.g., by zoning regulations). In this case the same functional is to be minimized, but a constraint $L - \bar{L} \leqslant 0$ is to be added, where $\bar{L}$ is exogeneously given. If, in addition to this constraint, R is assumed to be zero, the model of Solow and Vickrey is obtained and constant width configuration can be efficient for some values of L. Their model can also, of course, be interpreted as implying

$$R = 0 \quad \text{for} \quad x \leqslant \bar{L}; \qquad R = \infty \quad \text{for} \quad x > L.$$

References

1. E. S. Mills and D. M. De Ferranti, Market choices and optimum city size", *Amer. Econ. Rev., Papers Proc.*, **61** (1971), 340–345.
2. R. M. Solow and W. S. Vickrey, Land use in a long narrow city, *J. Econ. Theory* **3** (1971), 430–447.